INTEGRATION (10)

DEFINITE INTEGRATION

I $\int_a^b f(x)\,dx$

b ← Upper Limit

a ← Lower Limit

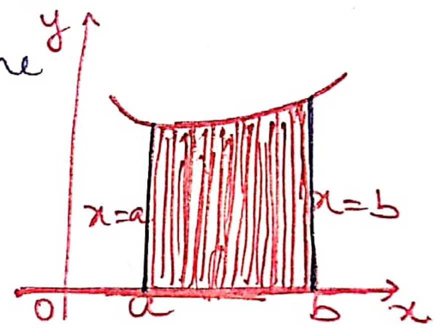
If $\int f(x)\,dx = F(x) + C$

then $\int_a^b f(x)\,dx = F(b) - F(a)$

(The value of definite integral is definite (fixed) free from constants)

II Geometric Representation of Definite Integrals

$\int_a^b f(x)\,dx$ = Area bounded by the curve $y = f(x)$, x axis and the straight lines $x = a$, $x = b$.



Q.1 $\int_0^{\pi/4} \sin x\,dx = -\left[\cos x\right]_0^{\pi/4} = -\left[\cos \pi/4 - \cos 0\right]$

$= -\frac{1}{\sqrt{2}} + 1 = 1 - \frac{1}{\sqrt{2}}$

Q.2 $\int_0^{\pi/4} \tan x \sec x\,dx = \left[\sec x\right]_0^{\pi/4} = \sec \pi/4 - \sec 0$

$= \sqrt{2} - 1$

Q. 3 $I = \int_0^1 x^2 e^x \, dx$

First of all we will solve indefinite integral.

$$\int \underset{I}{x^2} \underset{II}{e^x} dx = x^2 \int e^x dx - \int \left(\left(\frac{d}{dx} x^2\right) \int e^x dx \right) dx$$

$$= x^2 e^x - \int 2x . e^x dx$$

$$= x^2 e^x - 2\left[x \int e^x dx - \int \left(\frac{dx}{dx} \int e^x dx\right) dx\right]$$

$$= x^2 e^x - 2\left[x e^x - \int e^x dx\right]$$

$$= x^2 e^x - 2x e^x + 2e^x$$

$$\int_0^1 x^2 e^x dx = \left[x^2 e^x - 2x e^x + 2 e^x\right]_0^1$$

$$= \left[e - \cancel{2e} + \cancel{2e} - 0 + 0 - 2\right]$$

$$= e - 2$$

$$= e - 2 \quad \text{Ans}$$

Q.4 $I = \int_0^{\pi/2} \frac{\sin x}{1 + \cos^2 x} dx$

$$= -\int_1^0 \frac{dt}{1 + t^2}$$

$$= -\left[\tan^{-1} t\right]_1^0$$

$$= -\left[\tan^{-1} 0 - \tan^{-1} 1\right]$$

$$= -\left[0 - \pi/4\right]$$

$$= \pi/4$$

$\cos x = t$

$-\sin x dx = dt$

When $x = 0$

$\Rightarrow \cos x = \cos 0 = 1 = t$

When $x = \pi/2$

$t = \cos x = \cos \pi/2 = 0$

Q.5 $I = \int_0^{\pi/2} \frac{\cos x\,dx}{(1+\sin x)(2+\sin x)}$

Put $\sin x = t$

$\cos x\,dx = dt$

When $x = 0$

$t = \sin x = \sin 0 = 0$

When $x = \pi/2$

$t = \sin x = \sin \pi/2 = 1$

$= \int_0^1 \frac{dt}{(1+t)(2+t)}$

$= \int_0^1 \left[\frac{1}{1+t} - \frac{1}{2+t}\right] dt$

Rule of everywhere not here

$= \left[\log|1+t| - \log|2+t|\right]_0^1$

$= \left[\log 2 - \log 3\right] - \left[\log 1 - \log 2\right]$

$= \log\frac{2}{3} - \log\frac{1}{2}$

$= \log\frac{2}{3} \times 2$

$= \log\frac{4}{3}.$

Q.6 $I = \int_0^{\pi/2} \underset{I}{x^2} \underset{II}{\cos x}\,dx$

$I_1 = \int^{\pi/2}_0 x^2 \cos x\,dx$

$= x^2 \int \cos x\,dx - \int\left(\frac{d}{dx} x^2 \int \cos x\,dx\right)dx$

$= x^2 \sin x - \int 2x \sin x\,dx$

$= x^2 \sin x + 2\left[-\cos x \cdot x - \int \frac{dx}{dx}(-\cos x)\,dx\right]$

$= x^2 \sin x + 2x\cos x - 2\int \cos x\,dx$

$I = \left[x^2 \sin x + 2x\cos x - 2\sin x\right]_0^{\pi/2}$

$= \left[\frac{\pi^2}{4}\sin\pi/2 - 2\pi/2 \cos\pi/2 - 2\sin\pi/2\right] - \left[0 + 0 - 0\right]$

$= \pi^2/4 - 0 - 2 = \frac{\pi^2 - 8}{4} = \frac{\pi^2 - 8}{4}$