Differentiation

(1) $\frac{d}{dx}x^n = nx^{n-1}$

or

$\frac{d}{dx}x^{n+1} = (n+1)x^n$

(2) $\frac{d}{dx}e^x = e^x$

(3) $\frac{d}{dx}\sin x = \cos x$

(4) $\frac{d}{dx}\cos x = -\sin x$

(5) $\frac{d}{dx}\tan x = \sec^2 x$

(6) $\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$

(7) $\frac{d}{dx}\sec x = \sec x \tan x$

(8) $\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$

(9) $\frac{d}{dx}a^x = a^x \log_e a$

(10) $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$

(11) $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$

(12) $\frac{d}{dx}\cot^{-1} x = \frac{-1}{1+x^2}$

(12) $\frac{d}{dx}\sec^{-1} x = \frac{1}{x\sqrt{x^2-1}}$

$\frac{d}{dx}\operatorname{cosec}^{-1} x = -\frac{1}{x\sqrt{x^2-1}}$

Integration

(1) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

(2) $\int e^x dx = e^x + c$

(3) $\int \cos x\, dx = \sin x + c$

(4) $\int \sin x\, dx = -\cos x + c$

(5) $\int \sec^2 x\, dx = \tan x + c$

(6) $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$

(7) $\int \sec x \tan x\, dx = \sec x + c$

(8) $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + c$

(9) $\int a^x dx = \frac{a^x}{\log_e a} + c$

(10) $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$

$= -\cos^{-1} x + c$

(11) $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

$= -\cot^{-1} x + c$

(12) $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$

$= -\operatorname{cosec}^{-1} x + c$

Differentiation	Integration
(13) $\frac{d}{dx} \log x = \frac{1}{x}$	$\int \frac{1}{x} dx = \log x + c$

Indefinite Integrals.

(1) Let $\phi(x)$ be a function (फलन) s.t

$$\phi'(x) = f(x) \quad \text{or} \quad \frac{d}{dx}\phi(x) = f(x)$$

$$\Rightarrow \frac{d}{dx}[\phi(x) + c] = f(x)$$

$$\Longleftrightarrow \int f(x)\,dx = \phi(x) + c$$

∴ Integral is also called antiderivative.

(2) Standard Results of Integration

(i) $\int k\, f(x)\,dx = k\int f(x)\,dx$ where k is constant

(ii) $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

Practice Questions (Set I)

(1) $I = \int \left[x^3 + 5x^2 - 4 + \frac{7}{x} + \frac{2}{\sqrt{x}}\right] dx$

$$= \frac{x^4}{4} + \frac{5x^3}{3} - 4x + 7\log x + 2\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c \quad \left\{\because \int x^n\,dx = \frac{x^{n+1}}{n+1} + c\right\}$$

$$= \frac{x^4}{4} + \frac{5x^3}{3} - 4x + 7\log x + 4\sqrt{x} + c$$

(2) $I = \int \frac{x^3 + 5x^2 + 4x + 1}{x^2}\,dx$ {Rem: If Nr contains more than one term and Dr contains one term, divide}

$$= \int \left(x\,dx + 5 + \frac{4}{x} + \frac{1}{x^2}\right) dx$$

$$= \int x\,dx + \int 5\,dx + \int \frac{4}{x}\,dx + \int \frac{1}{x^2}\,dx$$

$$= \frac{x^2}{2} + 5x + 4\log x - \frac{1}{x} + c$$

Rem (1) $1+\cos 2x = 2\cos^2 x$ (2) $1-\cos 2x = 2\sin^2 x$ (2)

Q.3 $\int \frac{2}{1+\cos 2x}\,dx = \int \frac{2}{2\cos^2 x}\,dx = \int \sec^2 x$

$= \tan x + c$

Q.4 $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 dx$

$= \int \left[(\sqrt{x})^2 + 2\sqrt{x}\cdot\frac{1}{\sqrt{x}} + \left(\frac{1}{\sqrt{x}}\right)^2\right] dx$

$= \int \left[x + 2 + \frac{1}{x}\right] dx$

$= \frac{x^2}{2} + 2x + \log x + c$

Q.5 Rem $\int \sqrt{1+\sin 2x}\,dx$

$= \int \sqrt{\sin^2 x + \cos^2 x + 2\sin x\cos x}\,dx$ $\left\{\because \begin{array}{l}\sin^2 x + \cos^2 x = 1 \\ \sin 2A = 2\sin A\cos A\end{array}\right\}$

$= \int \sqrt{(\sin x + \cos x)^2}\,dx$

$= \int (\sin x + \cos x)\,dx$

$= -\cos x + \sin x + c$

Q.6 $\int \tan^2 x\,dx$

$= \int (\sec^2 x - 1)\,dx$ $\{\because \tan^2 x = \sec^2 x - 1\}$

$= \tan x - x + c$

Q.7

$$\int \frac{dx}{\sin^2 x \cos^2 x}$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\, dx$$

$$= \int \left[\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right] dx$$

$$= \int \left[\sec^2 x + \operatorname{cosec}^2 x\right] dx$$

$$= \tan x - \cot x + C$$

Q.8

$$\int \frac{1}{1+\sin x}\, dx$$

$$= \int \frac{1-\sin x}{(1+\sin x)(1-\sin x)}\, dx$$

$$= \int \frac{1-\sin x}{1-\sin^2 x}\, dx$$

$$= \int \frac{1-\sin x}{\cos^2 x}\, dx = \int \left(\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}\right) dx$$

$$= \int \sec^2 x\, dx - \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}\, dx$$

$$= \int \sec^2 x\, dx - \int \tan x \sec x\, dx$$

$$= \tan x - \sec x + C$$

Q. 9 $\int \frac{2^x + 3^x}{5^x} dx$

$= \int \left(\frac{2^x}{5^x} + \frac{3^x}{5^x}\right) dx$

$= \int \left[\left(\frac{2}{5}\right)^x + \left(\frac{3}{5}\right)^x\right] dx$

$= \frac{\left(\frac{2}{5}\right)^x}{\log_e(2/5)} + \frac{\left(\frac{3}{5}\right)^x}{\log_e(3/5)} + c$

Q. 10 $\int \left[e^{x \log a} + e^{a \log x} + e^{a \log a}\right] dx$

$= \int \left[e^{\log a^x} + e^{\log x^a} + e^{\log a^a}\right] dx$ $\{\because \log m^n = n \log m\}$

$= \int (a^x + x^a + a^a) dx$ $\{\because e^{\log_e x} = x\}$

$= \frac{a^x}{\log_e a} + \frac{x^{a+1}}{a+1} + a^a x + c$

ASSIGNMENT # 1 (Integration)

Questions	Answers
Q.1 $\int \frac{(1+x)^3}{\sqrt{x}}\,dx$	(1). $2\sqrt{x}+2x^{3/2}+\frac{6}{5}x^{5/2}+\frac{2}{7}x^{7/2}+C$
Q.2 $\int \sqrt{1-\cos 2x}\,dx$	(2) $-\sqrt{2}\cos x + C$
Q.3 $\int \frac{\cos 2x}{\sin^2 x\cos^2 x}\,dx$	(3) $-\cot x-\tan x+C$
Q.4 $\int \frac{1-\cos x}{1+\cos x}\,dx$	(4) $2(\operatorname{cosec} x-\cot x)-x+C$ or $2\tan x/2 - x + C$
Q.5 $\int \frac{(x+1)(x-2)}{\sqrt{x}}\,dx$	(5) $\frac{2}{5}x^{5/2}-\frac{2}{3}x^{3/2}-4\sqrt{x}+C$
Q.6 $\int \frac{\sin^2 x}{1+\cos x}\,dx$	(6) $x-\sin x+C$
Q.7 $\int \frac{e^{\log\sqrt{x}}}{x}\,dx$	(7) $2\sqrt{x}+C$
Q.8 $\int \sqrt{1-\sin 2x}\,dx$	(8) $\int$ $-\cos x+\sin x+C$
Q.9 $\int \frac{\operatorname{cosec} x}{\operatorname{cosec} x-\cot x}\,dx$ [Hint- Multiply & Divide by $\operatorname{cosec} x+\cot x$]	(9) $-\cot x-\operatorname{cosec} x+C$
Q.10 $\int \frac{\sin^3 x-\cos^3 x}{\sin^2 x\cos^2 x}\,dx$	(10) $\sec x+\operatorname{cosec} x+C$