

Conics Or Conic Sections शांकव या शंक परिच्छेप

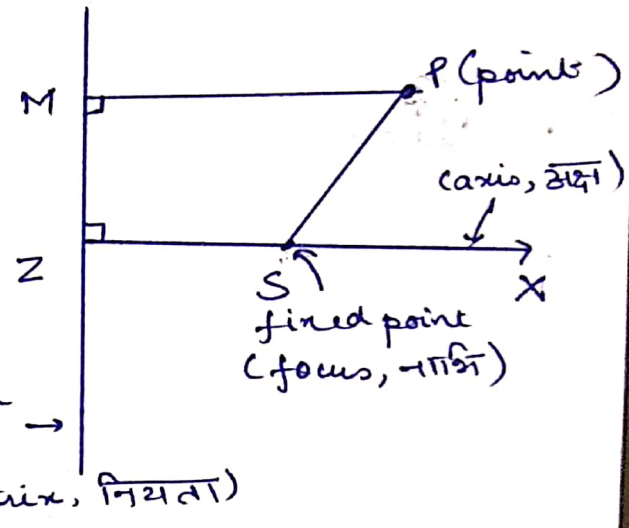
Def: शांकव ऐसे बिन्दु का बिन्दुपथ है, जो इस प्रकार गति करता है कि इसकी किसी स्थिर बिन्दु से दूरी तथा स्थिर रेखा से लंबागतक दूरी का अनुपात एक स्थिर राशि होती है।

Conic Section is locus of points which moves such that its distance from a fixed point bears a constant ratio to its distance from a fixed line.

$$\frac{PS}{PM} = e \text{ (eccentricity, उत्केन्द्रता)}$$

Types of Conics
(शांकव के प्रकार)

- ① If $e = 1$; Parabola (परवलय)
- ② If $e < 1$; Ellipse (दीर्घवृत्त)
- ③ If $e > 1$; Hyperbola (अतिपरवलय)



Formulae required to solve these questions.

①
$$\begin{matrix} A & B \\ (x_1, y_1) & (x_2, y_2) \end{matrix} \quad \text{दो बिन्दुओं के बीच दूरी} = \text{Distance b/w 2 points} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

② \perp Distance between a point and a line.

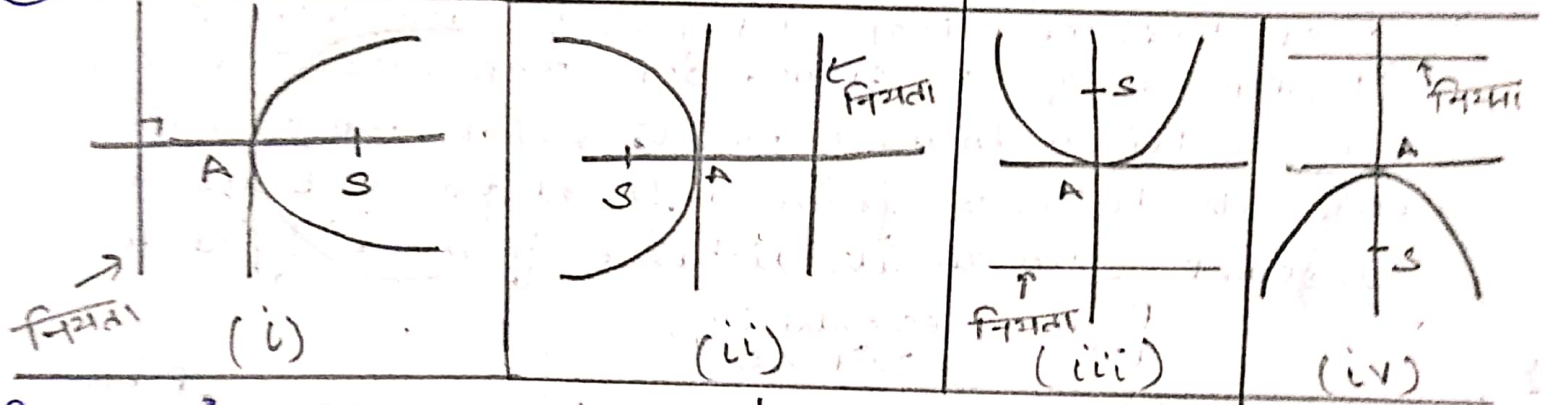
$$PD = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

The diagram shows a point P(x₁, y₁) and a line ax + by + c = 0. A perpendicular line segment PD is drawn from P to the line, representing the distance.

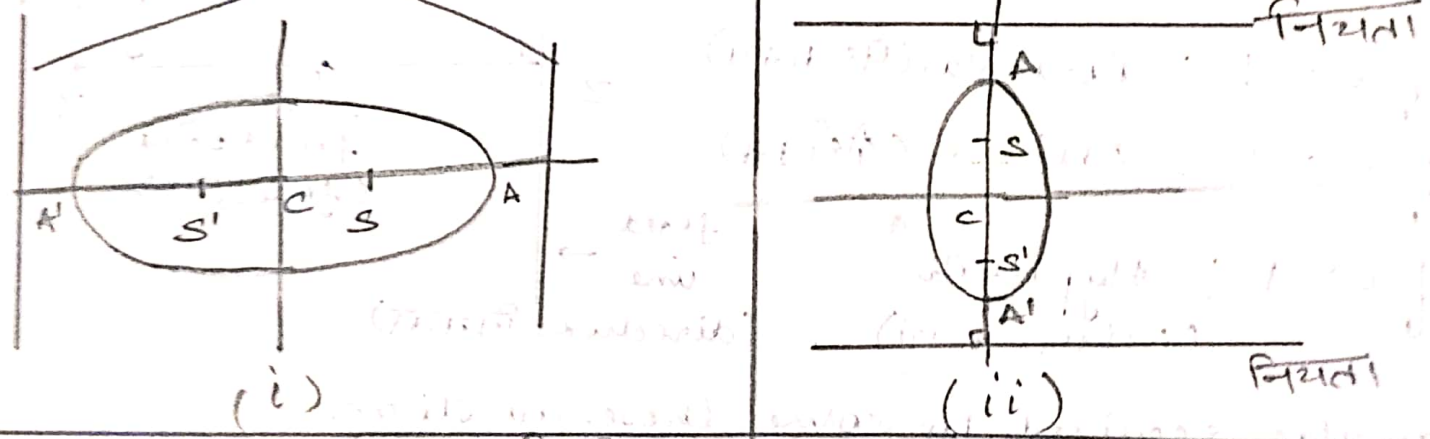
(2)

Basic figures of Parabola, Ellipse, Hyperbola
(परवलय), (दीर्घवृत्त), (अतिपरवलय)

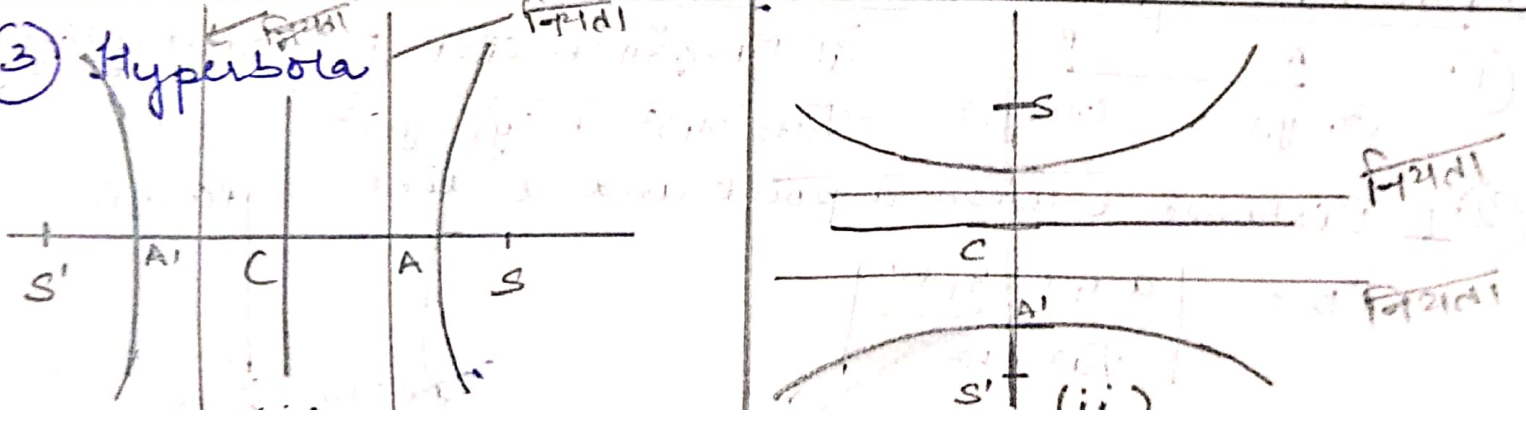
(1) Parabola



(2) Ellipse निम्नता



(3) Hyperbola



TYPE-1 Based on definition of Conic Sections

Q.1 उस परवलय का समीकरण ज्ञात करें जिसकी नाभि (5, 3) और नियता $3x - 4y + 1 = 0$ है।

Find equation of a parabola whose focus is at (5, 3) and directrix is $3x - 4y + 1 = 0$.

Soln. By definition of Conics

$$\frac{SP}{PM} = e = 1 \text{ (parabola)}$$

$$\Rightarrow SP = PM$$

$$\sqrt{(x-5)^2 + (y-3)^2} = \frac{|3x - 4y + 1|}{\sqrt{(3)^2 + (-4)^2}}$$

Squaring on both sides

$$(x-5)^2 + (y-3)^2 = \frac{(3x - 4y + 1)^2}{25}$$

$$25[x^2 - 10x + 25 + y^2 - 6y + 9] = 9x^2 + 16y^2 + 1 - 24xy - 8y + 6x$$

$$16x^2 + 9y^2 + 24xy - 256x - 142y + 849 = 0$$

Q.2 उस दीर्घवृत्त का समीकरण ज्ञात करें जिसकी नाभि (-1, 1), नियता $x - y + 3 = 0$ तथा इकेन्द्रता $\frac{1}{2}$ है।

Find equation of an ellipse with focus at (-1, 1) and directrix $x - y + 3 = 0$, eccentricity is $\frac{1}{2}$.

Soln. By definition of Conics

$$\frac{SP}{PM} = e$$

$$SP = ePM$$

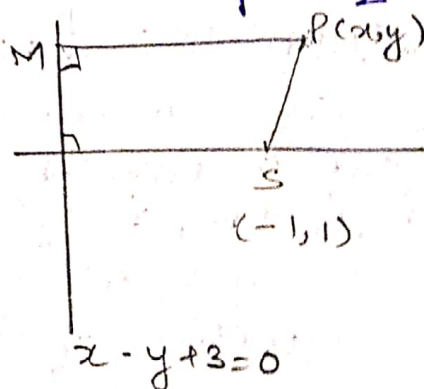
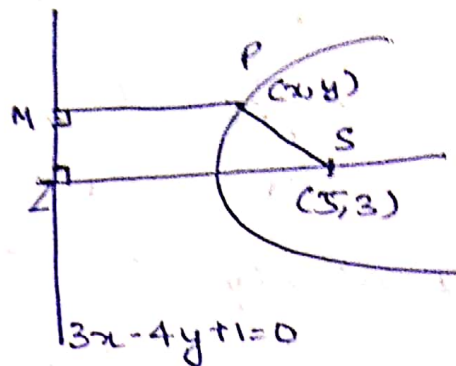
$$\sqrt{(x+1)^2 + (y-1)^2} = \frac{1}{2} \frac{|x - y + 3|}{\sqrt{(1)^2 + (-1)^2}}$$

On squaring

$$[(x+1)^2 + (y-1)^2] 4 \times 2 = (x - y + 3)^2$$

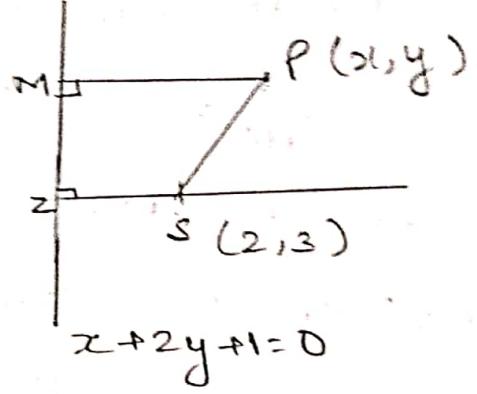
$$[x^2 + 2x + 1 + y^2 - 2y + 1] 8 = x^2 + y^2 + 9 - 2xy + 6x - 6y$$

$$7x^2 + 7y^2 + 2xy + 10x - 10y + 7 = 0$$



Q.3 इस अतिपरवलय का समीकरण ज्ञात करो जिसकी नियता $x+2y+1=0$, नाभि $(2,3)$ तथा अर्ध-प्रता $\sqrt{2}$ है।
 Find equation of a hyperbola with directrix as $x+2y+1=0$, focus $(2,3)$ and eccentricity $\sqrt{2}$.

Soln. $\frac{SP}{PM} = e$
 $\Rightarrow SP = e \cdot PM$



$$\sqrt{(x-2)^2 + (y-3)^2} = \sqrt{2} \frac{|x+2y+1|}{\sqrt{1^2+2^2}}$$

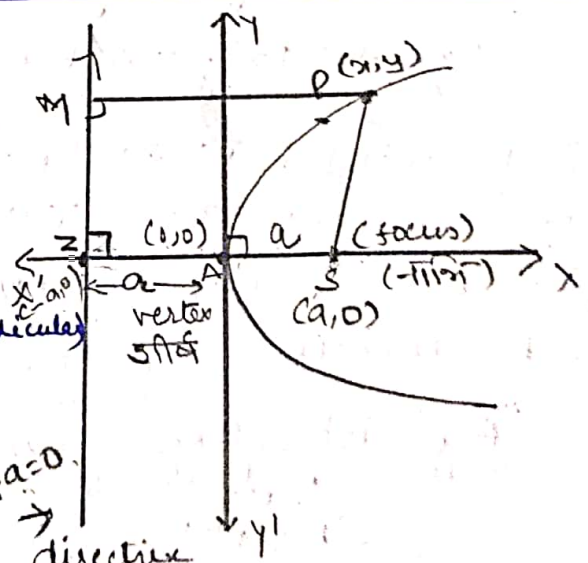
$$[(x-2)^2 + (y-3)^2] \cdot 5 = 2(x+2y+1)^2$$

$$[x^2 - 4x + 4 + y^2 - 6y + 9] \cdot 5 = 2(x^2 + 4xy + 4y^2 + 1 + 4x + 2y)$$

$$3x^2 - 3y^2 - 8xy - 24x - 38y - 63 = 0$$

Derivation of Standard Equation of Parabola

- ① परवलय का शीर्ष $A(0,0)$
Vertex $A(0,0)$
- ② नाभि $S(a,0)$; Focus $S(a,0)$
- ③ यहाँ नियता (Directrix) x अक्ष (axis) के लम्बवत (perpendicular) है, और y अक्ष के समानान्तर,
 \therefore नियता का समीकरण



$x = \text{constant}$
 क्योंकि परवलय पर स्थित किसी भी बिन्दु की दूरी नाभि और नियता से बराबर है
 अतः $AS = AZ = a$

\Rightarrow equation of directrix ; $x = -a$

By def. If P is any point on the parabola then $SP = PM$

$$\Rightarrow \sqrt{(x-a)^2 + (y-0)^2} = \frac{|x+a|}{\sqrt{1^2}}$$

On Squaring both sides

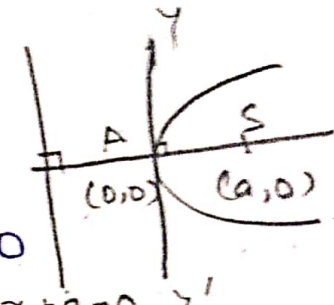
$$x^2 - 2ax + a^2 + y^2 = x^2 + a^2 + 2ax$$

$$\Rightarrow y^2 = 4ax$$

This is equation of right handed parabola.

Note I For Right Handed Parabola

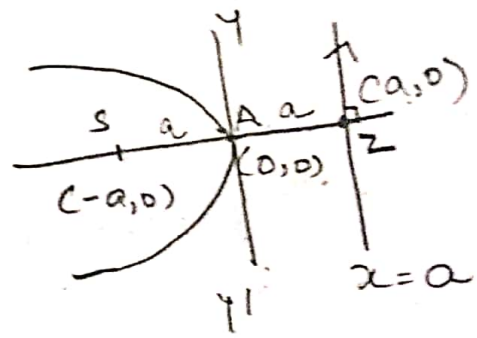
Eqn. $\rightarrow y^2 = 4ax$



- 1 नाभि (Focus) $(a, 0)$
- 2 शीर्ष (Vertex) $(0, 0)$
- 3 अक्ष (Axis) x axis ; $y=0$
- 4 नियता (Directrix) ; $x \neq a=0$ $x+a=0$
- 5 Tangent at vertex ; $x=0$
(शीर्ष पर स्पर्श)

II Left Handed Parabola

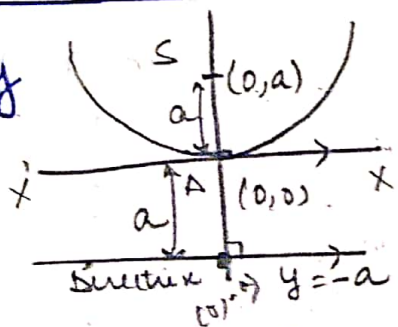
Eqn. $y^2 = -4ax$



- 1 नाभि (Focus) $(-a, 0)$
- 2 शीर्ष (Vertex) $(0, 0)$
- 3 अक्ष (Axis) ; $y=0$
- 4 नियता (Directrix) ; $x+a=0$
- 5 Tangent at vertex ; $x=0$

III Upward Parabola

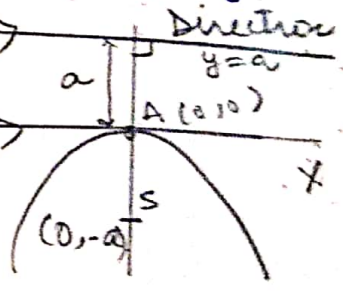
Eqn. $x^2 = 4ay$



- 1 नाभि (Focus) $(0, a)$
- 2 शीर्ष (Vertex) $(0, 0)$
- 3 अक्ष (Axis) ; y axis ; $x=0$
- 4 नियता (Directrix) ; $y = -a$
(parallel to x axis or $y+a=0$)
- 5 Tangent at vertex ; x axis
 $y=0$

IV Downward Parabola

Eqn $\rightarrow x^2 = -4ay$



- 1 नाभि (Focus) $(0, -a)$
- 2 शीर्ष (Vertex) $(0, 0)$
- 3 अक्ष (Axis) ; y axis ; $x=0$
- 4 नियता (Directrix) $y=a$
- 5 Tangent at vertex
 $y=0$